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ON THE STAGNATION-POINT IGNITION OF A PREMIXED COMBUSTIBLE

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Abstract—The ignitability of a cold premixed combustible at the stagnation point of an isothermal hot surface is analyzed in the limit of large activation energy. Analytical solutions were obtained for the first order perturbation, resulting in explicit expressions for the heat transfer at the wall, the temperature and species profiles, and most importantly an ignition criterion which states that ignition is expected to occur when a suitably-defined Damköhler number exceeds unity. It is further demonstrated that this state corresponds to the case of zero heat transfer from the wall, which has been intuitively used in the past as the ignition criterion.

NOMENCLATURE

a_i, reaction order of i;
A_0, A_1, coefficients in outer temperature expression, equation (30);
B, constant in reaction rate expression, equation (6);
C_p, specific heat;
D, diffusion coefficient;
E, activation energy;
f, stream function;
H, function defined in equation (21);
j, $j = 0$ and 1 for two-dimensional and axisymmetric flows;
n, temperature exponent in reaction rate expression;
p, pressure;
Q, heat release per unit mass of fuel consumed;
R, curvature of surface;
R^0, universal gas constant;
T, temperature;
T_a, activation temperature, E/R^0;
$\tilde{T}$, C_pT/Q;
u, v, x and y components of velocity;
W, molecular weight;
$\bar{W}$, average molecular weight;
x, y, physical co-ordinates;
X, mole fraction;
Y, mass fraction;
$\tilde{Y}_i$, $(\nu_F W_F/\nu_i W_i) Y_i$.

Greek symbols

α, coefficient for external flow, $u_\infty = \alpha x$;
β, heat-transfer parameter, $\tilde{T}_W - \tilde{T}_\infty$;
δ, defined in equation (27);
Δ, Damköhler number, equation (35);
ε, perturbation parameter, $\tilde{T}_W^2/\tilde{T}_a$;
η, similarity variable;
θ, temperature perturbation in inner region;
λ, thermal conductivity;
Λ, defined in equation (42);
μ, viscosity coefficient;
ν, stoichiometric coefficient, equation (5);
ξ, transformed independent variable, equation (20);
σ, stoichiometric oxidizer to fuel mass ratio;
ρ, density;
χ, stretched co-ordinate in inner region;
ω, reaction rate, equation (6).

Subscripts

f, frozen state;
i, indices for species;
in, out, inner and outer region;
F, O, P, fuel, oxidizer, and product;
$W, 0$, wall;
∞, ambiance.

I. INTRODUCTION

THERE have been both practical and fundamental interests in the study of the stagnation-point combustion of a premixed gaseous flow. On the practical side there are such important problems like flame-stabilization and the ignition of a flammable mixture by a hot projectile. On the fundamental side stagnation-point flow is one of the few chemically-reacting flow systems which admit similarity solutions. This then greatly facilitates mathematical manipulation and physical interpretation of the solutions.

Chambre [1] initiated the theoretical study on stagnation point ignition. A first-order, one-step overall chemical reaction is assumed. The ignition criterion used, however, is not the rigorous S-curve criterion [2, 3] but is rather an intuitively motivated one which states that ignition is expected to occur when there is sufficient chemical heat release in the gas phase such that the heat transfer from the hot stagnation point ceases, viz. $(\partial T/\partial y)_0 = 0$. Based on observations of the behavior of the solution for the non-reacting case and capitalizing on the fact that the non-dimensional activation energy is a large number, the system of equations was integrated across the boundary layer and an approximate expression governing the ignitability of the system was derived.

Sharma and Sirignano [4] obtained numerical solutions to the problem, assuming a second order reaction kinetics and stoichiometric concentrations of the ambient reactants. Non-unity Prandtl and Schmidt numbers were used. The ignition criterion used is again the state of adiabaticity used by Chambre.

Smith *et al.* [5] numerically obtained both the transient and the steady solutions, and demonstrated that the upper and lower branches of the S-curve are stable to small perturbations whereas the middle branch is always unstable. It is further shown that an unignited state of weak chemical activity can persist even though a temperature maximum exists in the gas phase, indicating that the adiabaticity criterion is not the correct ignition criterion.

Alkidas and Durbetaki [6, 7, 8] further investigated the steady-state system using numerical solutions. In particular it was shown [8] that the values of the wall temperatures for ignition obtained from the adiabaticity criterion and the S-curve criterion compare very favorably.

Saitoh [9] also numerically studied the same problem and showed the existence of a nearly constant, maximum local Damköhler number, at which the response curve changes from the monotonic, to the S-shaped, behavior. This state, however, is not the extinction limit as indicated in [9]. Rather it is the limit separating states in which transitions between nearly-frozen and nearly-equilibrium flows are smooth from those in which the transitions are abrupt as characterized by distinct ignition and extinction events.

In the present investigation we shall perform an asymptotic analysis for the steady-state system in the realistic limit of large activation energies. Only the weakly reactive and ignition states are studied herein. As will be shown the resulting governing equations are sufficiently simple such that analytical solutions are possible. This not only provides an explicit ignition criterion but also enables detailed exploration of the behavior of the system; in particular we shall demonstrate the existence of a close mathematical relationship between the adiabaticity and the S-curve criteria governing ignition.

II. GOVERNING EQUATIONS

The steady-state two-dimensional and axisymmetric boundary-layer flow for a chemically reacting mixture is governed by [4, 10],

Continuity:

$$\frac{\partial(\rho u R^j)}{\partial x} + \frac{\partial(\rho v R^j)}{\partial y} = 0. \tag{1}$$

Momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} - \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \rho_\infty u_\infty \frac{\mathrm{d}u_\infty}{\mathrm{d}x}. \tag{2}$$

Species:

$$\rho u \frac{\partial \tilde{Y}_i}{\partial x} + \rho v \frac{\partial \tilde{Y}_i}{\partial y} - \frac{\partial}{\partial y}\left(\rho D \frac{\partial \tilde{Y}_i}{\partial y}\right) = -\omega, \qquad i = 0, F. \tag{3}$$

Energy:

$$\rho u \frac{\partial \tilde{T}}{\partial x} + \rho v \frac{\partial \tilde{T}}{\partial y} - \frac{\partial}{\partial y}\left(\frac{\lambda}{C_p} \frac{\partial \tilde{T}}{\partial y}\right) = \omega \tag{4}$$

in which it has been assumed that the pressure and viscous heating terms are negligible in equation (4), that a single binary diffusion coefficient D exists for all species pairs, that the specific heat C_p is a constant, and that reactions between the fuel F and the oxidizer O leading to the formation of the product P can be represented by a one-step overall irreversible reaction

$$\nu_O[O] + \nu_F[F] \rightarrow \nu_P[P] \tag{5}$$

which has a reaction rate

$$\left(\frac{\omega}{\nu_F W_F}\right) = BT^n \left(\frac{X_O p}{R^0 T}\right)^{a_O} \left(\frac{X_F p}{R^0 T}\right)^{a_F} \mathrm{e}^{-E/R^0 T}. \tag{6}$$

By further assuming an ideal gas equation of state

$$p = \rho R^0 T / \bar{W}, \tag{7}$$

where $\bar{W}$ is the average molecular weight $\bar{W}^{-1} = \sum Y_i/W_i$, equation (6) can be written as

$$\omega = \left(\frac{B\nu_F W_F}{W_O^{a_O} W_F^{a_F}}\right)\left(\frac{p\bar{W}}{R^0}\right)^{(a_O + a_F)} T^{(n - a_O - a_F)} \times Y_O^{a_O} Y_F^{a_F} \mathrm{e}^{-E/R^0 T}. \tag{8}$$

It is significant to note that in equation (6) we have used quite a generalized representation of the reaction rate, which has arbitrary reaction orders with respect to the fuel and the oxidizer, and which depends on the temperature through both the Arrhenius factor and a power variation T^n. It will be shown subsequently that explicit solutions are obtained even with this generalized kinetic expression. Finally, we note that $R^j(x)$ describes the curvature of the surface such that $j = 0$ for two-dimensional flow and $j = 1$ for axisymmetric flow.

Equations (1)–(4) are to be solved subject to the boundary conditions

$$u(x, 0) = v(x, 0) = 0; \quad u(x, \infty) = u_\infty; \tag{9}$$

$$\tilde{T}(x, 0) = \tilde{T}_W; \quad \tilde{T}(x, \infty) = \tilde{T}_\infty; \tag{10}$$

$$\frac{\partial \tilde{Y}_i}{\partial x}(x, 0) = 0; \quad \tilde{Y}_i(x, \infty) = \tilde{Y}_{i\infty}; \tag{11}$$

where $\tilde{T}_W$ is a constant.

Using the conventional boundary-layer variables

$$s = \int_0^x \rho_\infty(x')\mu_\infty(x')u_\infty(x')R^{2j}(x')\,\mathrm{d}x' \tag{12}$$

$$\eta = u_\infty R^j (2s)^{-1/2} \int_0^y \rho(x, y')\,\mathrm{d}y' \tag{13}$$

and

$$f(s, \eta) = \psi(x, y)/(2s)^{1/2} \tag{14}$$

where the stream function $\psi(x, y)$ is defined by

$$\rho u R^j = \frac{\partial \psi}{\partial y}, \quad \rho v R^j = -\frac{\partial \psi}{\partial x} \tag{15}$$

such that the continuity equation is automatically satisfied, it can be shown [4] that at the stagnation point equations (2)–(4) are respectively transformed to

$$\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} + \frac{1}{2^j}\left[\left(\frac{\rho_\infty}{\rho}\right) - \left(\frac{df}{d\eta}\right)^2\right] = 0 \tag{16}$$

$$\frac{d^2 \tilde{Y}_i}{d\eta^2} + f\frac{d\tilde{Y}_i}{d\eta} = \frac{\omega}{2^j \alpha\rho} \tag{17}$$

$$\frac{d^2 \tilde{T}}{d\eta^2} + f\frac{d\tilde{T}}{d\eta} = -\frac{\omega}{2^j \alpha\rho} \tag{18}$$

with the boundary condition for equation (16) being

$$f(0) = f'(0) = 0, \quad f'(\infty) = 1. \tag{19}$$

In deriving equations (16)–(18) it has also been assumed that the product $\rho\mu$ is a constant, and that the Prandtl number $\mu C_p/\lambda$ and the Schmidt number $\mu/\rho D$ are both unity. The constant α characterizes the external flow through $u_\infty = \alpha x$.

By further defining a new independent variable

$$\xi = H(\eta)/H(\infty) \tag{20}$$

where

$$H(\eta) = \int_0^\eta \exp\left\{-\int_0^{\eta'} f(\eta'')\,d\eta''\right\} d\eta', \tag{21}$$

equations (17) and (18) can be equivalently represented as

$$\frac{d^2(\tilde{Y}_i + \tilde{T})}{d\xi^2} = 0 \tag{22}$$

and

$$\frac{d^2 \tilde{T}}{d\xi^2} = -\left[\frac{H(\infty)}{(dH/d\eta)}\right]^2 \frac{\omega}{2^j \alpha\rho} \tag{23}$$

where we have used the Shvab–Zeldovich formulation to eliminate the production term from one of the equations.

Equation (22) can be readily integrated, yielding an explicit relation between $\tilde{Y}_i$ and $\tilde{T}$ as

$$\tilde{Y}_i = \tilde{Y}_{i\infty} + \tilde{T}_\infty - (1-\xi)(d\tilde{T}/d\xi)_0 - \tilde{T}. \tag{24}$$

Therefore the final equations we have to solve are the coupled equations (16) and (23) subject to the boundary conditions equations (19) and

$$\tilde{T}(\xi = 0) = \tilde{T}_W, \quad \tilde{T}(\xi = 1) = \tilde{T}_\infty. \tag{25}$$

Equation (23) can also be written in expanded form as

$$\frac{d^2 \tilde{T}}{d\xi^2} = -\delta\left[\frac{H(\infty)}{(dH/d\eta)}\right]^2 \tilde{Y}_O^{a_O} \tilde{Y}_F^{a_F} \tilde{T}^{(n+1-a_O-a_F)} \times e^{-\tilde{T}_a/\tilde{T}} \tag{26}$$

where

$$\delta = \frac{B}{2^j \alpha} \frac{\nu_F W_F}{W_O^{a_O} W_F^{a_F}} \left(\frac{p\bar{W}}{R^0}\right)^{a_O + a_F - 1} \times \left(\frac{Q}{C_p}\right)^{(n+1-a_O-a_F)} \sigma^{a_O}, \tag{27}$$

$T_a = E/R^0$, $\sigma = \nu_O W_O/\nu_F W_F$, and $\tilde{Y}_i(\tilde{T})$ is given by equation (24).

III. ASYMPTOTIC ANALYSIS AND RESULTS

1. *Completely frozen solution*

In the limit of infinitely large activation energy, $\tilde{T}_a \to \infty$, the flow field will be completely frozen. The solution of equation (26) subject to equation (25) shows a linear temperature profile

$$\tilde{T}_f = \tilde{T}_W - \beta\xi \tag{28}$$

where $\beta = \tilde{T}_W - \tilde{T}_\infty$ is a heat-transfer parameter. Putting equation (28) in equation (24) reveals the obvious fact that the species concentrations are uniform throughout,

$$\tilde{Y}_{if} = \tilde{Y}_{i\infty}. \tag{29}$$

For large but finite values of $\tilde{T}_a$, chemical reactions are expected to initiate first near the hot wall since it has the highest temperature in the flow field. However, at a short distance away from the wall the slight decrease in temperature is sufficient to freeze the reactions due to the temperature-sensitive Arrhenius factor. Therefore the flow field is expected to consist of an inner, diffusive–reactive region next to the wall, and an outer, diffusive–convective region away from it. The characteristics of these two regions will be analyzed in the following.

2. *Outer solution*

In the outer region chemical reactions will be assumed to be frozen to all orders. Hence using the boundary condition at $\xi = 1$, equation (26) shows that the perturbed temperature profile is still linear, given by

$$\tilde{T}_{out} = \tilde{T}_\infty + A_0(1-\xi) + \varepsilon A_1(1-\xi) + O(\varepsilon^2) \tag{30}$$

where $\varepsilon = \tilde{T}_W^2/\tilde{T}_a$ is the small parameter of expansion for the present problem.

3. *Inner solution*

In the inner region weak chemical reactions take place and are responsible for the onset of the ignition event. Let the inner variable be $\chi = \beta\xi/\varepsilon$ and the inner temperature distribution $\tilde{T}_{in}$ be slightly perturbed from its frozen value by an amount $\varepsilon\theta(\chi)$ such that

$$\tilde{T}_{in} = \tilde{T}_W + \varepsilon[\theta(\chi) - \chi] + O(\varepsilon^2). \tag{31}$$

Equation (24) then shows that the species distribution in the inner region is given by

$$\tilde{Y}_i = \tilde{Y}_{i\infty} - \beta(d\theta/d\chi)_0 + O(\varepsilon). \tag{32}$$

Hence putting equations (31), (32) and the relation

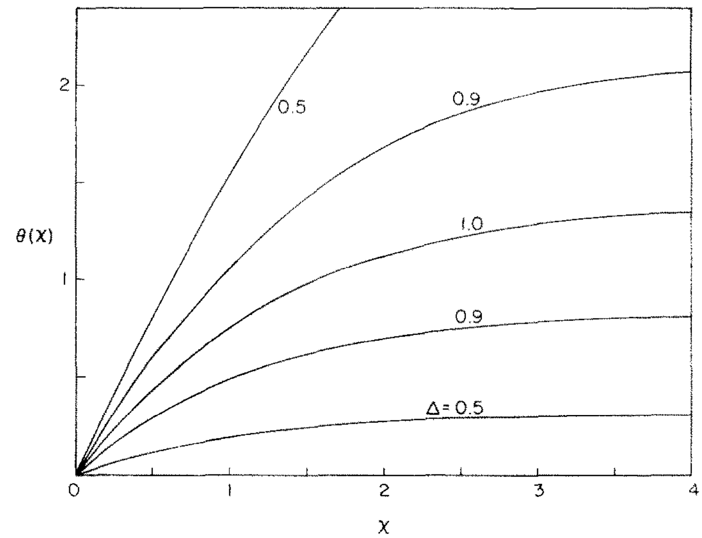

FIG. 1. Temperature perturbation in the inner region for different values of the Damköhler number Δ.

(Appendix A)

$$dH/d\eta = 1-O(\varepsilon^3) \quad \text{as} \quad \eta \to 0 \tag{33}$$

into equation (26), we get

$$\frac{d^2\theta}{d\chi^2} = -\frac{\Delta}{2}e^{(\theta-\chi)} \tag{34}$$

where

$$\Delta = \left(\frac{2\delta\varepsilon}{\beta^2}\right)H_f^2(\infty)\left[\tilde{Y}_{O\infty}-\beta\left(\frac{d\theta}{d\chi}\right)_0\right]^{aO} \times\left[\tilde{Y}_{F\infty}-\beta\left(\frac{d\theta}{d\chi}\right)_0\right]^{aF}\tilde{T}_W^{(n+1-aO-aF)}e^{-\tilde{T}_a/\tilde{T}_W} \tag{35}$$

is the appropriate Damköhler number for the present problem. It includes both the conventionally-defined first and second Damköhler numbers, and is the proper representation for the ratio of the chemical effect to transport effect such that $\Delta \to 0$ under situations of vanishing chemical activity.

It may be noted that because of equation (33) and the appearance of only $H_f(\infty)$ in equation (35) (Appendix A), the solution of equation (34) is completely independent of the momentum equation equation (16). Furthermore, the evaluation of $H_f(\infty)$ requires only the zeroth order solution of $f(\eta)$, which is governed by equation (16) and

$$\frac{d^2\tilde{T}_f}{d\eta^2}+f\frac{d\tilde{T}_f}{d\eta} = 0 \tag{36}$$

with the boundary conditions equations (19) and (25). Fortunately this system of equations has been numerically solved in detail [11]. In particular, the parameter $H_f(\infty)$, which is the only $f(\eta)$ related quantity required for the present ignition analysis, is simply given by

$$H_f(\infty) = -\frac{\beta}{(d\tilde{T}_f/d\eta)_0} \tag{37}$$

and has been extensively tabulated [11].

Equation (34) is to be solved subject to the boundary condition at the wall

$$\theta(0) = 0 \tag{38}$$

and a second one obtained through matching with the outer solution. This will now be performed.

4. *Matching and final solution*

Matching the outer and inner solutions in the limit of $\chi \to \infty$, we have

$$\lim_{\chi\to\infty} \tilde{T}_\infty + (A_0+\varepsilon A_1)(1-\varepsilon\chi/\beta) = \lim_{\chi\to\infty} \tilde{T}_W+\varepsilon[\theta(\chi)-\chi].$$

Hence to the zeroth order we have

$$A_0 = \beta$$

as should be. To the first order of matching we have

$$\theta(\infty) = A_1 \tag{39}$$

and

$$\left(\frac{d\theta}{d\chi}\right)_\infty = 0. \tag{40}$$

Therefore equation (34) is to be solved subject to equations (38) and (40), with the perturbation to the frozen temperature distribution in the outer region given by equation (39).

The detailed solution of equation (34) is carried out in Appendix B, from which it is shown

$$\theta(\chi) = \chi+\ln\left\{\frac{1}{\Delta}\left[1-\left(\frac{\Lambda e^{\pm\chi}-1}{\Lambda e^{\pm\chi}+1}\right)^2\right]\right\} \tag{41}$$

where

$$\Lambda = \frac{1+(1-\Delta)^{1/2}}{1-(1-\Delta)^{1/2}}. \tag{42}$$

Figure 1 shows some typical curves of equation (41).

It is seen that for $\Delta < 1$ two solutions exist whereas for $\Delta > 1$ no solution exists. This behavior indicates the occurrence of some critical phenomena, viz. ignition for the present case, when $\Delta = 1$, as will be further elaborated in the following.

5. *Response curve and ignition criterion*

From equation (41) it can be easily shown that

$$\theta(\infty) = \ln\left\{\frac{4}{\Delta}\left[\frac{1 \mp (1-\Delta)^{1/2}}{1 \pm (1-\Delta)^{1/2}}\right]\right\} \qquad (43)$$

which is plotted in Fig. 2. It is obvious that we have obtained the lower half of the S-shaped response curve.

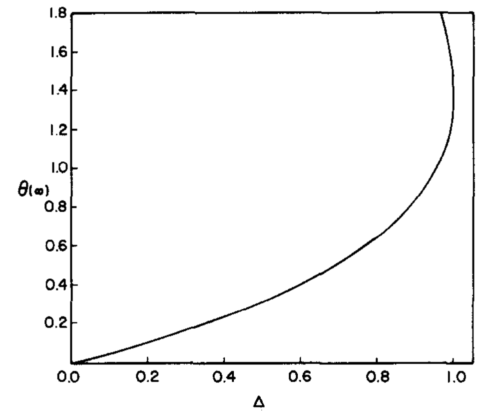

FIG. 2. Response curve showing the maximum perturbed temperature as a function of the Damköhler number Δ.

Hence the lower branch in Fig. 2, given by the positive root of equation (41), represents states with weak chemical activities. At $\Delta = 1$, the slope of the response curve becomes infinite and ignition is expected to occur. The system then instantaneously assumes an active burning mode represented by the upper branch of the S-curve which is not obtained here. The middle branch of the S-curve, given by the negative roots of equation (41), represents unstable states.

It is also of interest to investigate the heat transfer from the wall. From equation (41),

$$\left(\frac{d\theta}{d\chi}\right)_0 = 1 \mp (1-\Delta)^{1/2}. \qquad (44)$$

Therefore using equation (31) we have

$$\left(\frac{d\tilde{T}_{in}}{d\chi}\right)_0 = \mp (1-\Delta)^{1/2}. \qquad (45)$$

Hence during the weakly burning case there is always heat transfer from the wall to the gas. At the ignition point $\Delta = 1$ the heat transfer from the wall ceases. This result demonstrates that to first order of accuracy the adiabaticity criterion for ignition [1, 4] is exactly the S-curve ignition criterion. It also explains the close agreement in the ignition states obtained by using both criteria [8]. The phenomenon of having an unignited state with temperature maximum occurring in the gas phase [5] is then strictly a second order effect. This observation is probably of a general nature and hence applicable to systems other than the stagnation point flow.

An explicit ignition criterion can be obtained by substituting $\Delta = 1$ into equation (35), yielding

$$\left(\frac{2\delta\varepsilon}{\beta^2}\right) H_f^2(\infty)(\tilde{Y}_{O\infty} - \beta)^{a_O}(\tilde{Y}_{F\infty} - \beta)^{a_F} \times \tilde{T}_W^{(n+1-a_O-a_F)} e^{-\tilde{T}_a/\tilde{T}_W} \geqslant 1 \qquad (46)$$

for systems in which ignition is expected to occur.

Finally, it may be noted that for $(\tilde{Y}_{i\infty} - \beta) = O(\varepsilon)$, the first order terms in equation (32) also have to be included in the analysis. For such dilute systems, however, the response curve is expected to be monotonic and not exhibiting any distinct ignition event.

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APPENDIX A

Evaluations of $H(\eta)$ *related parameters*

First we need to evaluate

$$\frac{dH}{d\eta} = \exp\left\{-\int_0^\eta f(\eta')\,d\eta'\right\} \quad \text{as} \quad \xi \to 0.$$

To satisfy the requirements that $f(0) = f'(0) = 0$ and that $f''(0) \neq 0$ [12], we assume

$$f(\eta) \sim \eta^2 \quad \text{as} \quad \eta \to 0,$$

then

$$\int_0^\eta f(\eta')\,d\eta' \sim \eta^3,$$

and

$$\frac{dH}{d\eta} \simeq 1 - O(\eta^3).$$

But since

$$\eta = \varepsilon\chi H(\infty)/\beta,$$

therefore

$$\frac{dH}{d\eta} = 1 - O(\varepsilon^3).$$

Next we need to expand $H(\infty)$. Let $f(\eta) = f_f(\eta) + \varepsilon f_1(\eta) + O(\varepsilon^2)$, then

$$\begin{aligned} H(\infty) &= \int_0^\infty \exp\left\{-\int_0^\eta f(\eta')\,d\eta'\right\} d\eta \\ &\simeq \int_0^\infty \exp\left\{-\int_0^\eta f_f(\eta')\,d\eta'\right\} \exp\left\{-\varepsilon\int_0^\eta f_1(\eta')\,d\eta'\right\} d\eta \\ &\simeq \int_0^\infty \left[1 - \varepsilon\int_0^\eta f_1(\eta')\,d\eta'\right] \exp\left\{-\int_0^\eta f_f(\eta')\,d\eta'\right\} d\eta \\ &= \int_0^\infty \exp\left\{-\int_0^\eta f_f(\eta')\,d\eta'\right\} d\eta + O(\varepsilon) \\ &= H_f(\infty) + O(\varepsilon). \end{aligned}$$

APPENDIX B

Derivation of the inner solution

We aim to solve

$$d^2\theta/d\chi^2 = -(\Delta/2)\,e^{(\theta-\chi)} \tag{B1}$$

subject to

$$\theta(0) = 0, \quad (d\theta/d\chi)_\infty = 0. \tag{B2}$$

Let $\theta_1 = \theta - \chi$, then equations (B1) and (B2) become

$$d^2\theta_1/d\chi^2 = -(\Delta/2)\,e^{\theta_1} \tag{B3}$$

$$\theta_1(0) = 0, \quad (d\theta_1/d\chi)_\infty = -1. \tag{B4}$$

Integrating equation (B3) once and using the boundary condition at $\chi = \infty$, we obtain

$$(d\theta_1/d\chi) = \mp[1 - \Delta\,e^{\theta_1}]^{1/2}. \tag{B5}$$

Equation (B5) can be readily integrated by making a further transformation with $\theta_2 = (1 - \Delta\,e^{\theta_1})^{1/2}$. Integration of the resulting equation, using the boundary condition at $\chi = 0$, finally yields

$$\theta(\chi) = \chi + \ln\left\{\frac{1}{\Delta}\left[1 - \left(\frac{\Lambda\,e^{\mp\chi} - 1}{\Lambda\,e^{\pm\chi} + 1}\right)^2\right]\right\} \tag{B6}$$

SUR LE POINT D'ARRET D'IGNITION D'UN COMBUSTIBLE PREMELANGE

Résumé—On analyse la possibilité d'ignition d'un combustible prémélangé et froid au point d'arrêt d'une surface chaude isotherme, dans le cas d'une forte énergie d'activation. Des solutions analytiques sont obtenues pour une perturbation du premier ordre, sous une forme explicite, pour le transfert thermique à la paroi, les profils de température et d'espèces et un critère d'ignition qui établit que l'ignition apparait lorsqu'un nombre de Damköhler convenablement défini dépasse l'unité. On montre ensuite que cet état correspond au cas d'un transfert thermique nul à la paroi, ce qui a été intuitivement supposé dans le passé comme le critère d'ignition.

STAUPUNKTZÜNDUNG EINES BRENNBAREN GEMISCHES

Zusammenfassung—Die Zündfähigkeit eines kalten Brenngasgemisches am Staupunkt einer heißen isothermen Want wird im Bereich hoher Aktivierungsenergie untersucht. Analytische Lösungen wurden für den Störungsansatz 1. Ordnung erhalten, welche zu expliziten Ausdrüken für den Wärmeübergang an der Wand, die Temperatur-und Konzentrationsprofile und vor allen Dingen zu einem Zünd-Kriterium führten, welches aussagt, daß Zündung dann zu erwarten ist. wenn eine in geeigneter Form definierte Damköhler-Zahl den Wert eins überschreitet. Weiter wird gezeigt, daß dieser Zustand mit dem Fall verschwindender Wärmeübertragung von der Wand zusammenfällt, der schon in der Vergangenheit intuitiv als Zündbedingung angesehen worden ist.

О ВОСПЛАМЕНЕНИИ ПРЕДВАРИТЕЛЬНО ПЕРЕМЕШАННОЙ ГОРЮЧЕЙ СМЕСИ В КРИТИЧЕСКОЙ ТОЧКЕ

Аннотация — В работе анализируется воспламеняемость холодной предварительно перемешанной горючей смеси в критической точке изотермически нагретой поверхности для больших значений энергии активации. Получены аналитические решения для возмущений первого порядка, которые позволяют установить в явном виде выражения для теплопереноса на стенке, профилей температуры и концентрации смеси и, что наиболее важно, для критерия воспламеняемости, свидетельствующего о том, что воспламенение происходит в том случае, когда значение соответствующим образом определенного числа Дамкёлера больше единицы. Показано также, что это состояние соответствует случаю отсутствия теплопереноса от стенки, используемого ранее в качестве критерия воспламеняемости.